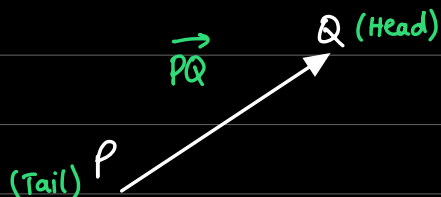
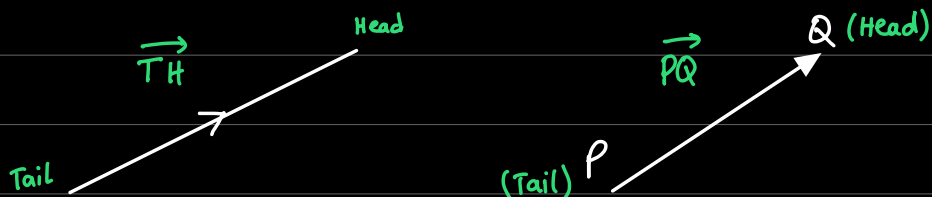


$\vec{AB}$  vector

$\overline{AB}/AB$  Line

$\widehat{AB}$  arc

## VECTORS (5-6 Marks)



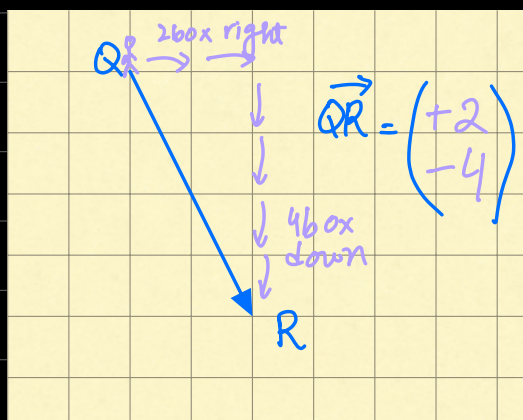
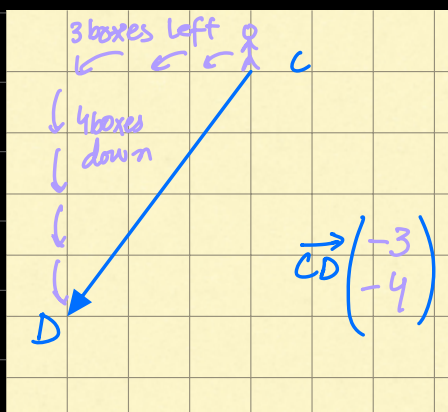
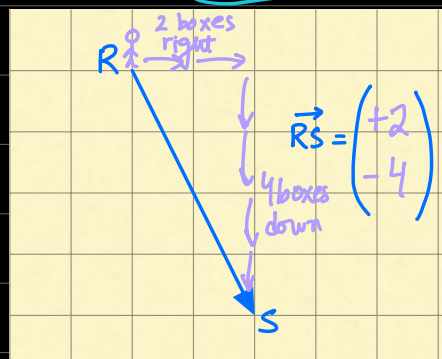
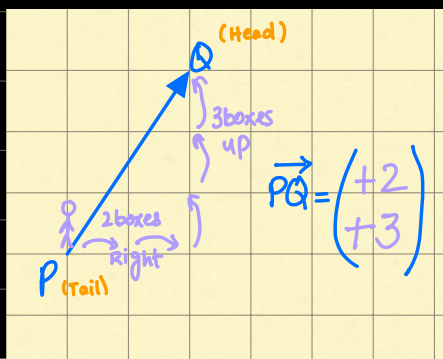
## COLUMN VECTOR (always on grid)

$\begin{pmatrix} x \\ y \end{pmatrix}$   $\begin{matrix} \oplus & \ominus \\ \text{right} & / \text{Left} \end{matrix}$   
 $\begin{matrix} \oplus & \ominus \\ \text{up} & / \text{down} \end{matrix}$

Mr Vector!



I will start from tail and go to head.  
I can only move in 4 directions  
up/down/left/right  
I cannot move slant



## MAGNITUDE (LENGTH) OF VECTOR:

If you have a vector  $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$

Symbol: |name of vector|

THEN, MAGNITUDE (LENGTH) IS :

$$|\vec{AB}| = \sqrt{x^2 + y^2}$$

$$\begin{aligned} &(-4)^2 \\ &(-4)(-4) \\ &+ 16 \end{aligned}$$

Q:  $\vec{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ , Find  $|\vec{AB}|$

$$\begin{aligned} |\vec{AB}| &= \sqrt{(3)^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Note: When we put values in any formula, use BRACKETS.

Q:  $\vec{AB} = \begin{pmatrix} 6 \\ q \end{pmatrix}$ , Given that  $|\vec{AB}| = 10$

Find values of  $q$ .

$$\begin{aligned} |\vec{AB}| &= \sqrt{(6)^2 + (q)^2} \\ (10)^2 &= (\sqrt{36 + q^2})^2 \\ 100 &= 36 + q^2 \\ 100 - 36 &= q^2 \\ q^2 &= 64 \end{aligned}$$

Caution:  $\frac{\sqrt{b^2 + q^2}}{\sqrt{b^2} + \sqrt{q^2}}$

You cannot open power separately in ADD/SUB

Whenever we take square root to cancel square, we introduce  $\pm$  sign on side with square root.

$$q = \pm \sqrt{64}$$

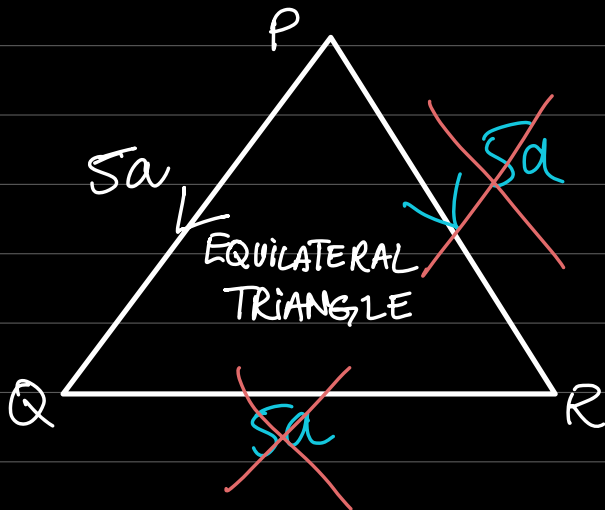
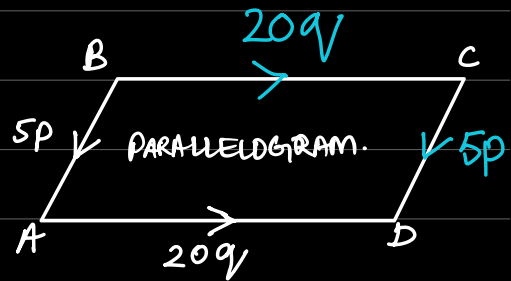
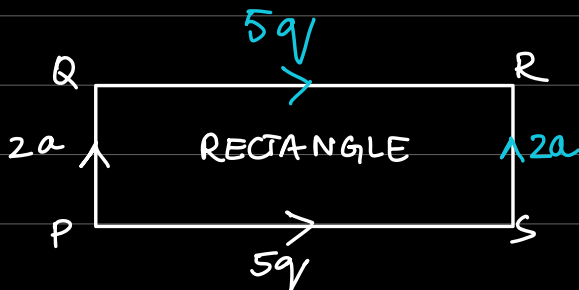
$$q = \pm 8$$

Exception: TRIG, mensuration.

Most imp  $\rightarrow$  **EQUAL VECTORS**

MUST SATISFY TWO CONDITIONS:-

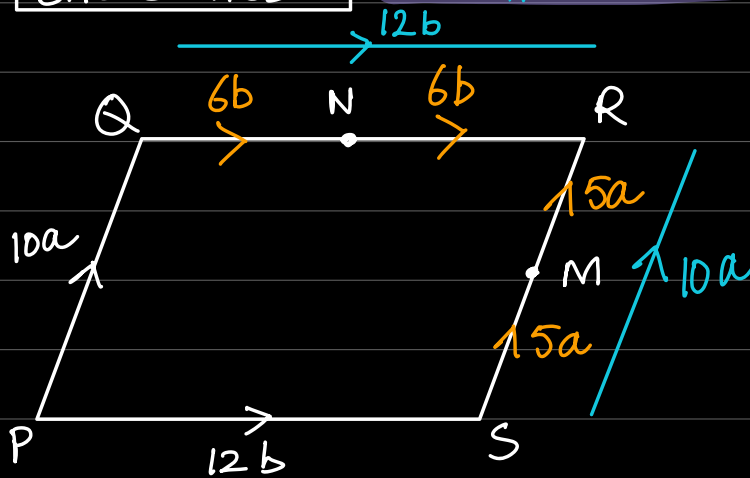
- 1) EQUAL LENGTH.
- 2) PARALLEL TO EACH OTHER.



There are no equal vectors here because to be equal the lines must be PARALLEL.

## BASIC MODE

The +/- signs rule for U/D/L/R are not applied here.

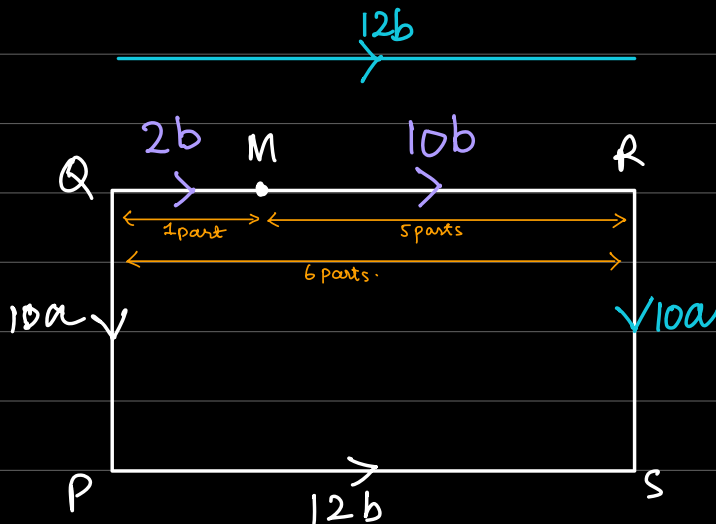


PQRS is a parallelogram.  
M and N are midpoints.

Find:

- (a)  $\vec{PR} = +12b + 10a$   
 $= +10a + 12b$
- (b)  $\vec{QS} = +12b - 10a$   
 $= -10a + 12b$
- (c)  $\vec{PN} = +10a + 6b$
- (d)  $\vec{PM} = 12b + 5a$
- (e)  $\vec{NS} = 6b - 10a$
- (f)  $\vec{MN} = 5a - 6b$

## ADVANCED MODE (RATIO METHOD)



PQRS is a RECTANGLE.

$$QM = \frac{1}{5} MR$$

Find:

- $\vec{PR} = 12b - 10a$
- $\vec{QS} = 10a + 12b$
- $\vec{QM} = 2b$
- $\vec{PM} = -10a + 2b$
- $\vec{MS} = 10b + 10a$
- $\vec{RM} = -10b$

Ratio Method:-

$$QM = \frac{1}{5} MR$$

STEPS: 1) Correct form.

$$\frac{QM}{MR} = \frac{1 \text{ parts}}{5 \text{ parts.}}$$

2) mark parts on the diagram

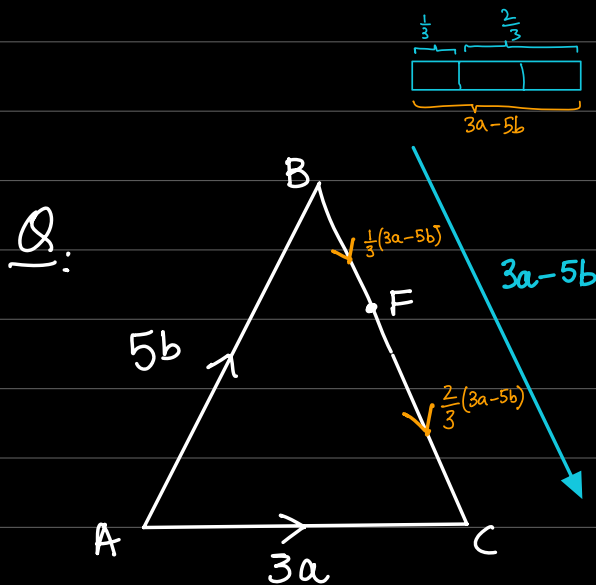
| 3)      |            | PARTS | ACTUAL |
|---------|------------|-------|--------|
| (GIVEN) | $\vec{QR}$ | 6     | $12b$  |
| (FIND)  | $\vec{QM}$ | 1     | $x$    |

$$6x = 12b \times 1$$

$$x = \frac{12b}{6}$$

$$x = 2b$$

$$\vec{QM} = 2b$$



Given that F is such that

$$BF = \frac{1}{3} BC$$

Note for Shortcut in Ratio Method:

IF ONE OF THE VECTORS IN RATIO ARE KNOWN, JUST PUT THAT VALUE IN THE RATIO. NO NEED TO APPLY FULL THREE STEP RATIO METHOD

(i) Find  $\vec{BC} = -5b + 3a = 3a - 5b$

EXAM TIP: It is better to write +ve terms before -ve terms.

It helps us in workings of next parts.

(ii) Find  $\vec{BF}$

$$BF = \frac{1}{3} BC$$

$$\vec{BF} = \frac{1}{3} (3a - 5b)$$

(iii) Find  $\vec{AF}$ .

$$\vec{AF} = \frac{5b \times 3}{1 \times 3} + \frac{1}{3} (3a - 5b)$$

$$\vec{AF} = \frac{15b + 3a - 5b}{3}$$

$$\vec{AF} = \frac{3a + 10b}{3}$$

### WORKING RULES:

1) EQUAL VECTORS

2) RATIO METHOD

(Always check for shortcut)

(Shortcut: If you know value of one of vectors in ratios, do not apply full ratio method).

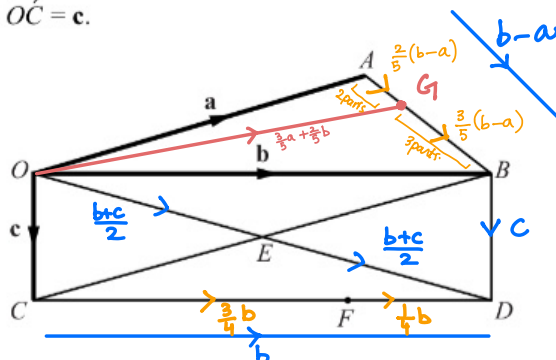
Super  
IMP

3) Label each vector that you find on diagram. (Don't draw new lines)

4) Whenever you have fractions, take LCM.

In a difficult Question  $\begin{cases} \text{Fraction} \rightarrow \text{LCM} \\ \text{Label everything.} \end{cases}$

- 8  $OAB$  is a triangle and  $OBDC$  is a rectangle where  $OD$  and  $BC$  intersect at  $E$ .  
 $F$  is the point on  $CD$  such that  $CF = \frac{3}{4} CD$ .  
 $\vec{OA} = \mathbf{a}$ ,  $\vec{OB} = \mathbf{b}$  and  $\vec{OC} = \mathbf{c}$ .



$$\vec{CF} = \frac{3}{4} \vec{CD}$$

$$\vec{CF} = \frac{3}{4} \mathbf{b}$$

- (a) Express, as simply as possible, in terms of one or more of the vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ ,

(i)  $\vec{AB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

Answer .....  $\mathbf{b} - \mathbf{a}$  [1]

(ii)  $\vec{OE} = \frac{\vec{OD}}{2} = \frac{\mathbf{b} + \mathbf{c}}{2}$

Answer .....  $\frac{\mathbf{b} + \mathbf{c}}{2}$  [1]

(iii)  $\vec{EF} = \frac{2(\mathbf{b} + \mathbf{c})}{2 \times 2} - \frac{1}{4} \mathbf{b}$

$$\vec{EF} = \frac{2\mathbf{b} + 2\mathbf{c} - \mathbf{b}}{4} = \frac{\mathbf{b} + 2\mathbf{c}}{4}$$

Answer .....  $\frac{\mathbf{b} + 2\mathbf{c}}{4}$  [2]

- (b)  $G$  is the point on  $AB$  such that  $\vec{OG} = \frac{3}{5} \mathbf{a} + \frac{2}{5} \mathbf{b}$ .

- (i) Express  $\vec{AG}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .  
 Give your answer as simply as possible.

$$\vec{AG} = -\frac{\mathbf{a}}{1 \times 5} + \frac{3}{5} \mathbf{a} + \frac{2}{5} \mathbf{b} = \frac{-5\mathbf{a} + 3\mathbf{a} + 2\mathbf{b}}{5} = \frac{-2\mathbf{a} + 2\mathbf{b}}{5} = \frac{2\mathbf{b} - 2\mathbf{a}}{5} = \frac{2(\mathbf{b} - \mathbf{a})}{5}$$

Answer .....  $\frac{2}{5}(\mathbf{b} - \mathbf{a})$  [1]

- (ii) Find  $AG : GB$ .

Answer .....  $2 : 3$  [1]

- (iii) Express  $\vec{FG}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ .  
 Give your answer as simply as possible.

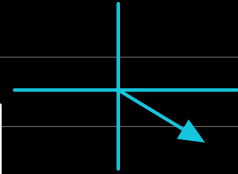
$$\vec{FG} = \frac{1}{4} \mathbf{b} - \frac{\mathbf{c}}{5} - \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$\vec{FG} = \frac{5\mathbf{b} - 2\mathbf{c} - 12(\mathbf{b} - \mathbf{a})}{20}$$

$$= \frac{5\mathbf{b} - 2\mathbf{c} - 12\mathbf{b} + 12\mathbf{a}}{20} = \frac{12\mathbf{a} - 7\mathbf{b} - 2\mathbf{c}}{20}$$

Answer .....  $\frac{12\mathbf{a} - 7\mathbf{b} - 2\mathbf{c}}{20}$  [2]

## POSITION VECTORS

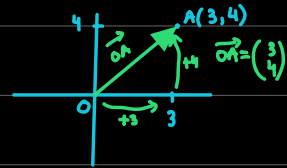


NAME STARTS WITH "O"

O = origin (0,0)

$\vec{OP}$  = Position vector of P

$\vec{OA}$  = Position vector of A



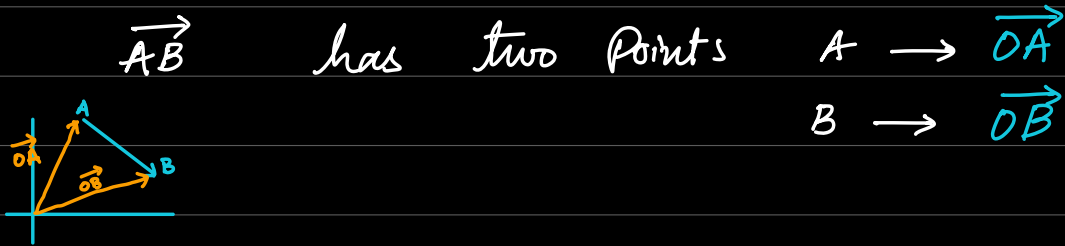
[ POSITION VECTOR  
OF A POINT ] = [ COORDINATES  
OF THAT POINT ]

$\vec{OA} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$  means A (3, 5)

C (7, 5) means  $\vec{OC} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

WHENEVER YOU FEEL THAT  
THERE IS A QUESTION MIXING  
VECTORS and COORDINATE GEOMETRY  
IT IS ACTUALLY  
POSITION VECTOR.





$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$\vec{CX} = \vec{OX} - \vec{OC}$$

### EXAM QUESTION

Q:  $A(1, 3)$  and  $\vec{AB} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$   
 $\vec{OA} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Find coordinates of B.

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\begin{pmatrix} 7 \\ -3 \end{pmatrix} = \vec{OB} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \vec{OB}$$

$$\vec{OB} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \longrightarrow B(8, 0)$$

Q.  $C(5, 7)$  and  $\vec{AC} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$   
 $\hookrightarrow \vec{OC} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

Find coordinates of A.

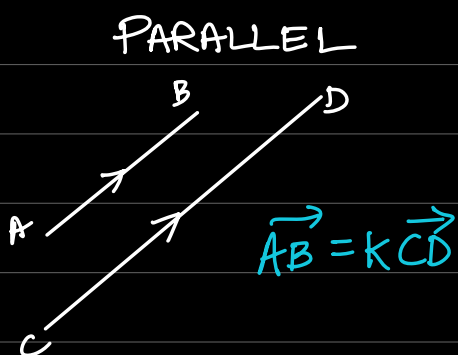
$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$\begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \vec{OA}$$

$$\vec{OA} + \begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

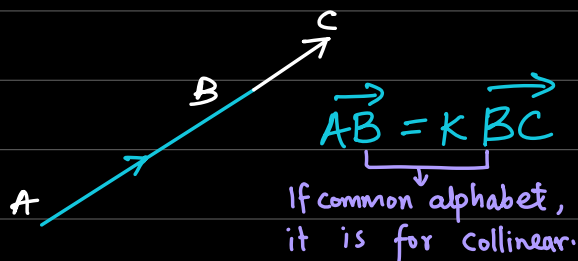
$$\vec{OA} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\vec{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \longrightarrow A(2, -3)$$



$\hat{=}$

COLLINEAR  
(on the same line)



$$\underset{\substack{\downarrow \\ \text{First} \\ \text{vector}}}{A} = \underset{\substack{\downarrow \\ \text{Just a} \\ \text{number.}}}{k} \underset{\substack{\downarrow \\ \text{Second} \\ \text{vector}}}{B}$$

**EXAM TIP** If the question already contains  $(k)$ , use another alphabet in our formula.

eg:  $A = tB$ ,  $A = xB$

Q:  $\vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$  ,  $\vec{CD} = \begin{pmatrix} 9 \\ k \end{pmatrix}$

Given that AB and CD are parallel, Find k.

$A = \cancel{X} B$

$A = t B$

$\begin{pmatrix} 9 \\ k \end{pmatrix} = t \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 9 \\ k \end{pmatrix} = \begin{pmatrix} 3t \\ 5t \end{pmatrix}$

$3t = 9$  ,  $k = 5t = 5(3)$   
 $t = 3$  ,  $\boxed{k = 15}$

Take the first vector which has the unknown. It keeps the workings simple.

Trigger  $\begin{cases} \text{Parallel} \\ \text{collinear} \\ \text{(on the same line)} \end{cases}$

Allowed Shortcut:

$\vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$   $\vec{CD} = \begin{pmatrix} 9 \\ k \end{pmatrix}$

$\frac{3}{9} = \frac{5}{k}$

$3k = 45$

$\boxed{k = 15}$

Shortcut working (Allowed in exam)

IF

PARALLEL,  
COLLINEAR  
ON THE SAME LINE

Q:  $\vec{AB} = 7a + 12b$   $\vec{BD} = 21a + (t-1)b$

Given that A, B and D lie on same line.  
Find t. (1 mark)

$$\frac{7a}{21a} = \frac{12b}{(t-1)b}$$

$$\frac{7}{21} = \frac{12}{t-1}$$

$$t-1 = 36$$

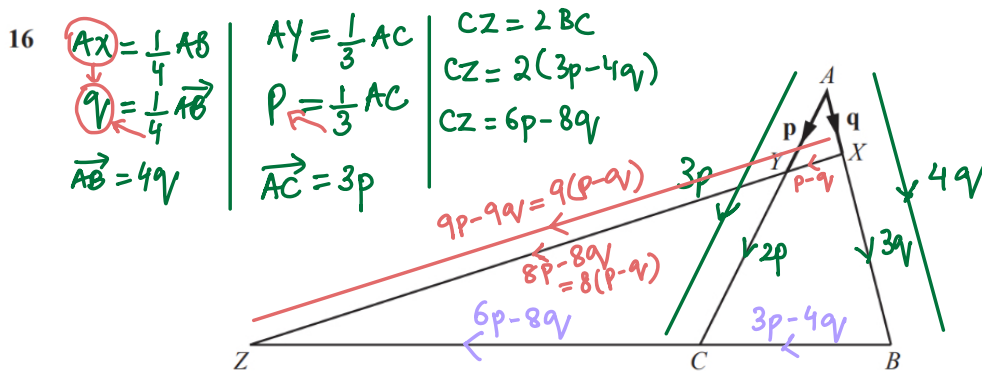
$$t = 37$$

Vectors:

- 1- Column Vectors + Magnitude.
- 2- Equal vectors, Ratio method, Traffic flow diagrams.
- 3- Position vectors
- 4- Parallel/collinear.

Notes  
Revise + 10 Questions  
with help  
from video + 10-15 Q  
Test  
yourself.

Questions regarding area ratios in triangles with vectors is related to SIMILARITY.



In the diagram,

$X$  is the point on  $AB$  where  $AX = \frac{1}{4}AB$ ,

$Y$  is the point on  $AC$  where  $AY = \frac{1}{3}AC$ ,

$Z$  is the point on  $BC$  produced where  $CZ = 2BC$ .

$\vec{AY} = \mathbf{p}$  and  $\vec{AX} = \mathbf{q}$ .

(a) Express, as simply as possible, in terms of  $\mathbf{p}$  and  $\mathbf{q}$ ,

(i)  $\vec{XY} = -\mathbf{q} + \mathbf{p} = \mathbf{p} - \mathbf{q}$

Answer  $\vec{XY} = \mathbf{p} - \mathbf{q}$  [1]

(ii)  $\vec{BC} = -4\mathbf{q} + 3\mathbf{p} = 3\mathbf{p} - 4\mathbf{q}$

Answer  $\vec{BC} = 3\mathbf{p} - 4\mathbf{q}$  [1]

(iii)  $\vec{XZ} = 3\mathbf{q} + 3\mathbf{p} - 4\mathbf{q} + 6\mathbf{p} - 8\mathbf{q} = 9\mathbf{p} - 9\mathbf{q}$

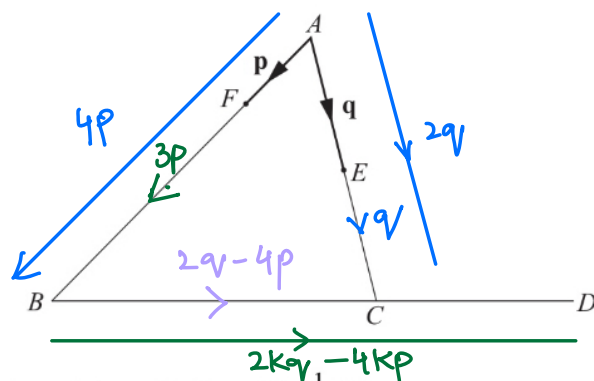
Answer  $\vec{XZ} = 9\mathbf{p} - 9\mathbf{q}$  [2]

(b) Hence find  $XY : YZ = \frac{XY}{YZ} = \frac{\mathbf{p} - \mathbf{q}}{8\mathbf{p} - 8\mathbf{q}} = \frac{\mathbf{p} - \mathbf{q}}{8(\mathbf{p} - \mathbf{q})} = \frac{1}{8} = 1 : 8$

$XY = \mathbf{p} - \mathbf{q}$

$YZ = 8\mathbf{p} - 8\mathbf{q}$

Answer  $1 : 8$  [1]



$$AF = \frac{1}{4} AB$$

$$p = \frac{1}{4} AB$$

$$AB = 4p$$

In the diagram,  $F$  is the point on  $AB$  where  $AF = \frac{1}{4} AB$ .

$E$  is the midpoint of  $AC$ .

$\vec{AF} = \mathbf{p}$  and  $\vec{AE} = \mathbf{q}$ .

(a) Express, in terms of  $\mathbf{p}$  and  $\mathbf{q}$ ,

(i)  $\vec{FE} = -\mathbf{p} + \mathbf{q} = \mathbf{q} - \mathbf{p}$

Answer  $\mathbf{q} - \mathbf{p}$  [1]

(ii)  $\vec{BC} = -4\mathbf{p} + 2\mathbf{q} = 2\mathbf{q} - 4\mathbf{p}$

Answer  $2\mathbf{q} - 4\mathbf{p}$  [1]

(b)  $D$  is the point on  $BC$  extended such that  $BD = kBC$ .

(i) Express  $\vec{FD}$  in terms of  $k$ ,  $\mathbf{p}$  and  $\mathbf{q}$ .

$$\vec{FD} = 3\mathbf{p} + 2k\mathbf{q} - 4k\mathbf{p}$$

$$= 3\mathbf{p} - 4k\mathbf{p} + 2k\mathbf{q}$$

$$\vec{FD} = (3 - 4k)\mathbf{p} + 2k\mathbf{q}$$

$$\begin{aligned} \vec{BD} &= k\vec{BC} \\ \vec{BD} &= k(2\mathbf{q} - 4\mathbf{p}) \\ \vec{BD} &= 2k\mathbf{q} - 4k\mathbf{p} \end{aligned}$$

Answer [1]

(ii) Given that  $F$ ,  $E$  and  $D$  are collinear, find the value of  $k$ .

$$\begin{aligned} \vec{FE} &= -\mathbf{p} + \mathbf{q} \\ \vec{FD} &= (3 - 4k)\mathbf{p} + 2k\mathbf{q} \end{aligned}$$

$$\begin{aligned} \frac{-1}{(3 - 4k)} &= \frac{1}{2k} \\ \frac{-1}{3 - 4k} &= \frac{1}{2k} \end{aligned}$$

$$\begin{aligned} -2k &= 3 - 4k \\ 4k - 2k &= 3 \\ 2k &= 3 \\ k &= \frac{3}{2} \end{aligned}$$

Answer  $k = \frac{3}{2}$  [2]